

M.Phil. MATHEMATICS PROGRAMME

SYLLABUS

2009-2010 onwards



PONDICHERY UNIVERSITY

PUDUCHERRY – 605 014

M.PHIL MATHEMATICS

Regulations

Eligibility for admission:

A candidate for admission into M.Phil. programme shall have studied M.Sc. Mathematics under 10 + 2 + 3 + 2 pattern of study.

Candidates who have secured 55% of marks or above in Master's Degree in Mathematics are eligible to apply.

Duration of study:

The course duration shall normally be of one year spread over two semesters. The maximum duration to complete the course shall be 3 years.

Medium:

The medium of instruction shall be English.

Passing minimum:

Passing Eligibility and Classification for the award of the Degree are as per the norms of the Choice Based Credit System.

Credit Requirements

A candidate of M.Phil. programme has to take 2 hard core courses and sufficient number of soft core courses and carry out a dissertation work.

The Course Work shall be done for a minimum of 18 credits. The Dissertation and Viva-voce carry 15 and 3 credits, respectively.

COURSE PATTERN

List of Hard Core Courses

Sl. No.	Course Code	Title of the Course	No. of Credits
1	MATH-605	Hydrodynamic Stability	4
2	MATH-619	Algebra	4

List of Soft Core Courses

Sl. No.	Course Code	Title of the Course	No. of Credits
1	MATH-602	Topics in Analysis	4
2	MATH-621	Theory of Graphs	4
3	MATH-622	Analysis	4
4	MATH-635	Algebraic Theory of Numbers	4
5	MATH-636	Integrable Systems	4
6	MATH-637	Lie Groups of Transformations and Differential Equations	4
7	MATH-638	Homological Algebra	4
8	MATH-639	Representation and Characters of Finite Groups	4

SYLLABUS

HARD CORE: MATH-605: HYDRODYNAMIC STABILITY (4 Credits)

UNIT-I

Theory of Stability – definition of stability and asymptotic – stability – reduction of equilibrium solution – elementary types of rest points – nodes – focus – centre – saddle points.

UNIT-II

General theorems of Lyapunov – Chetaev's instability theorem – examples.

UNIT-III

Rayleigh problem – Squire's theorem – Rayleigh's equation – Rayleigh's inflection points theorem – Fjortoft's theorem.

UNIT-IV

Lin's formula, Tollmien's inviscid solutions – Heisenberg's expansions – Reynolds stress – formula for variation of Reynolds stress.

UNIT-V

Howard's theorem – Unbounded flows – asymptotic formula for instability – piece wise linear profiles – Arnold's theorems.

Text books

1. Elsgolts, Differential Equations and Calculus of Variations (For units I and II)
2. Drazin and Reid, Hydrodynamic Stability (For units III and IV)
3. Arnold and Khesin, Topological methods in Hydrodynamics – Applied Mathematical Sciences 125 – Springer Verlag (For unit V)

HARD CORE: MATH - 619 ALGEBRA (4 Credits)

UNIT - I

Prime ideals, maximal ideals, Nil radical, Jacobson radical, Operation on ideals, Extension and contraction.

UNIT - II

Operation on sub-modules, Direct sum and product. Finitely generated modules, Exact sequences, Tensor product, Restriction and extension of Scalars.

UNIT - III

Rings and Modules of Fraction and Primary decomposition Local properties Extended and contracted Primary decomposition.

UNIT – IV

Integral dependence and chain conditions.

UNIT - V

Noetherian rings and Artinian rings

Text book

M.F Atiyah and I.G. Macdonald, Introduction to Commutative Algebra, Addison-Wesley.

Reference books

1. H. Matsumura, Commutative Ring Theory, Cambridge University Press.
2. I. Kaplansky, Commutative Rings.
3. O. Zariski and P. Samuel, Commutative Algebra.

SOFT CORE: MATH - 602 TOPICS IN ANALYSIS (4 Credits)

UNIT-I

Nets and sequences – Characterization of topological properties in terms of nets – Inadequacy of sequences – Subnets – Unconditionally summable series – Examples – Topological vector spaces – Balanced and absorbing sets – Locally convex spaces. The metric space $L_p[0, 1]$ for $0 < p < 1$.

UNIT-II

Finite dimensional topological vector spaces - Linear homomorphism with K^n (scalar field R or C) - Minkowski functional - Sublinear functionals and semi norms - Bounded and totally bounded sets - Metrizable topological vector spaces - Characterization of normable locally convex spaces.

UNIT-III

Continuous linear functionals and dual of locally convex spaces – Mazur's separation theorem in a topological vector space – First and second separation theorems.

UNIT-IV

Weak topology induced by a subset of the dual - Weak and weak* topologies - Weak and weak* convergent sequences – Comparison of weak, weak* and norm topologies – Reflexive spaces – Metrization of weak and weak* topologies on bounded sets – Polar sets.

UNIT-V

Bipolar theorem - Weak* continuous linear functionals on dual normed linear spaces - Goldstein's theorem – Banach-Alaoglu's theorem - Characterization of reflexive spaces as spaces with weakly compact unit balls – Closed subspaces of reflexive spaces are reflexive.

Reference books

1. R.E.Megginson, An Introduction to Banach Space Theory.
2. J. Conway, A Course on Functional Analysis.
3. H.H. Schefer, Topological Vector Spaces.
4. M.Fabian, P.Habala, P.Hajek, V.M. Santalucia, J. Pelant and V.Zizler, Functional Analysis and Infinite Dimensional Geometry, CMS Books in Mathematics, Springer Seminar Volume, 2001.

SOFT CORE: MATH - 621 THEORY OF GRAPHS (4 Credits)

UNIT-1:

Graphs and simple graphs - Special graphs (complete graphs, complement of graphs) - Graph isomorphism - Sub graphs - vertex degrees - Walks, paths, cycles - Graph connection and components - Bipartite graphs – Trees – Cutedges – Cutvertices - Blocks.

UNIT-II:

Matchings - Berge's theorem - Hall's theorem and its applications - Coverings in graphs - Konig's theorem - Perfect matching - Tutt's theorem and its applications.

UNIT-III:

Independent sets - Gallai's theorem - Ramsay numbers - Ramsay graphs - Erdos and Szekers theorem.

UNIT-IV:

Erdos theorem - Turan's theorem and its applications - Chromatic number of graphs - Critical graphs - Brook's theorem - Chromatic polynomials.

UNIT-V:

Plane and planar graphs - The five colour theorem - The four colour Theorem (statement only) - Directed graphs - Directed paths – Tournaments - Directed Hamilton paths and cycles.

Text book:

1. J.A.Bondy and U.S.R.Murthy, Graph Theory with Applications, 1976.

Reference books

1. F.Harary, Graph Theory, 1969.
2. M.Behzad and G.Chartrand, Introduction to the Theory of Graphs, 1971.
3. G.Chartrand and L.Lesniak, Graphs and Digraphs, 1979.
4. D.B.West, Introduction to Graph Theory.
5. K.R.Parthasarathy, Basic Graph Theory, 1994.
6. N.Deo, Graph Theory with Applications to Engineering and Computer Science, 1974.
7. R.J.Wilson, Introduction to Graph Theory, 1972.
8. L.R.Foulds, Graph Theory Applications, 1993.

SOFT CORE: MATH - 622 ANALYSIS (4 Credits)

Fourier Analysis

UNIT - I

Fourier transforms on \mathbb{R} - Fourier transform of rapidly decreasing functions -
Integration on Product spaces.

UNIT - II

Convolutions - Uniqueness of Fourier transforms - - Fourier inversion theorem -
The Plancherel theorem.

UNIT - III

Translation Invariant subspaces of $L^2(\mathbb{R})$. - The Banach algebra $L^1(\mathbb{R})$ – The complex
homomorphisms of $L^1(\mathbb{R})$.
Elementary theory of Banach algebras.

UNIT - IV

Elementary properties - Resolvent and spectrum - Spectral radius formula.

UNIT - V

Ideals and homomorphisms - Gelfand transforms - Gelfand theorem.

Reference books

1. W. Rudin, Real and Complex Analysis, TMH Edition, Second Edition, New York, 1962.
2. W. Rudin, Functional Analysis, TMH Edition, 1974
3. K. Yosida, Functional Analysis, Springer – Verlag, 1968.
4. E.H.Lieb and M. Loss, Analysis, Narosa Publishing House, New Delhi, 1997.

SOFT CORE: MATH - 635 ALGEBRAIC THEORY OF NUMBERS (4 Credits)

UNIT - I

Divisibility in principal ideal rings - The Diophantine equation $X^2 + Y^2 = Z^2$ - The diophantine equation $X^4 + Y^4 = Z^4$ - Euler's phi function - Modules over principal ideal rings - Roots of unity α in a field.

UNIT - II

Elements integral over a ring - Integrally closed rings - Elements algebraic over a field - Algebraic extensions - Conjugate elements - Conjugate fields - Integers in quadratic fields.

UNIT - III

Noetherian rings and modules - Application of integral elements - Properties of ideals - Dedekind rings.

UNIT - IV

Discrete sub groups of \mathbb{R} - Canonical imbedding of a number field - Finiteness of the ideal class group.

UNIT - V

The unit theorem - Units in imaginary quadratic fields - Units in real quadratic fields.

Reference book

P. Samuel, Algebraic Theory of Numbers, Houghton Mifflin Company, Boston, 1970.

SOFT CORE: MATH - 636 INTEGRABLE SYSTEMS (4 Credits)

UNIT -1

Introduction to linear and nonlinear waves – Korteweg - de Vries equation - Lax pairs – Conservation laws – Hamiltonian structures.

UNIT -2

Hirota's bilinear method – Korteweg - de Vries and KP equations - Finding three soliton solutions.

UNIT -3

Lie symmetry analysis - Heat, Burger's, Korteweg - de Vries and modified Korteweg – de - Vries equation and similarity reductions.

UNIT -4

Introduction to difference equations-Discrete heat, Burgers' and Korteweg – de –Vries.

UNIT -5

Applications of REDUCE and MAPLE mathematical softwares to certain problems in integrable systems.

Reference books

1. P. G. Drazin, Solitons, Cambridge University Press, Cambridge, 1983.
2. M. Lakshmanan and S. Rajesekhar, Nonlinear Dynamics - Integrability, Chaos and Patterns, Springer-Verlag, Berlin, 2003.
3. G. Bluman and S. Kumei, Symmetries and Differential Equations, Springer-Verlag, Berlin, 1989.
4. Kosmann - Schawarzba , B. Gramaticcos and K. M. Tamizhmani, (Ed.), Integrability to Nonlinear Systems, LNP 49, Springer – Verlag, Berlin, 1997.
5. R. Hirota. Mathematics of Solitons; Direct Method, Iwami, Japan, 1992.
6. D. Takahashi, Discrete and Ultra - discrete Systems, Kyoritsu, Japan, 2003.

SOFT CORE: MATH - 637
LIE GROUPS OF TRANSFORMATIONS AND DIFFERENTIAL EQUATIONS
(4 Credits)

UNIT -1

Introduction – Lie groups of transformation – Infinitesimal transformations.

UNIT -2

Extended group transformations and infinitesimal transformations (one independent – one dependent and two independent – two dependent).

UNIT -3

Lie algebras and applications.

UNIT -4.

Invariance of first and second order differential equations.

UNIT -5.

Invariance of partial differential equations of first and second order – Elementary examples.

Reference book

G. W. Bluman and S. Kumei, Symmetries and Differential Equations, Springer Verlag, 1989.

Unit 1: Chapter 2 (omit sections 1-6, 14)

Unit 2: Chapter 3 (omit sections 1-3, 6, 7, 10, 11)

Unit 3: Chapter 4 (omit sections 1,6-13)

Unit 4: Chapter 5 (omit sections 1,5-10)

Unit 5: Chapter 6 (omit sections 1,5-7)

SOFT CORE: MATH - 638 HOMOLOGICAL ALGEBRA (4 Credits)

UNIT -1

Free modules - Projective modules - Injective modules - Flat modules.

UNIT -2

Complexes and derived functors.

UNIT -3

Ext and Tor functors.

UNIT -4

Homological dimensions.

UNIT -5

Tensor algebra, symmetric algebra and exterior algebra.

Reference books

1. J.Rotman, An Introduction to Homological Algebra, Academic Press, 1979.
2. N.S. Gopalakrishnan, Commutative Algebra, Oxonian Press Pvt. Ltd., 1984.
3. S.D. Dand and R.M. Forbe, Abstract Algebra, Prentice Hall International.

SOFT CORE: MATH – 639
REPRESENTATION AND CHARACTERS OF FINITE GROUPS (4 Credits)

UNIT -1

Modules - Tensor products - Restricted and induced modules- Indecomposable modules - Completely reducible module - Schur lemma- Semi simple rings - The Jacobson radical - Group algebras - Maschke's theorem.

UNIT -2

Artinian and Noetherian rings - Structure theory of ring – Density theorem - Wedderburn-Artin theorem for semi simple rings.

UNIT -3

Linear and matrix representation – Equivalent and irreducible representations - Induced and restricted representation - Tensor product and inner products of representation.

UNIT -4

Representations of the symmetric groups - Young subgroups -Tableaux - Tabloids - Specht modules - Standard tableaux- Branching rule.

UNIT - 5

RSK Algorithm - The Hook length formula - Increasing and decreasing subsequences - Group characters.

Text books

1. C.W. Curtis and I.Reiner, Representation Theory of Finite Groups and Associative Algebras, Inter Science Publishers, 1962 (Chapters 2, 4 and 5).
2. B.E.Sagan, The Symmetric Group, Second Edition, Springer International Edition, Springer-Verlag, New York, 2001 (Chapters 2 and 3).

Reference books

1. W.Fulton and J.Harris, Representation Theory - A First Course, International Edition, Springer-Verlag, New York, 2004.
2. Jacobson, Basic Algebra II, Hindustan Publishing Corporation (India), 1983.
3. I.N Herstein, Non-Commutative Rings, Mathematical Association of America.

