

M.Sc. MATHEMATICS PROGRAMME

SYLLABUS

2009-2010 onwards



PONDICHERY UNIVERSITY

PUDUCHERRY – 605 014

M.Sc. MATHEMATICS PROGRAMME

Regulations

Eligibility for admission:

A candidate for admission into M.Sc. programme shall have studied B.Sc. Mathematics under 10 + 2 + 3 pattern of study.

Candidates who have secured 55% of marks or above in Bachelor's Degree in Mathematics are eligible to apply.

Duration of study:

The course duration shall normally be of two years spread over four semesters. The maximum duration to complete the course shall be 4 years.

Medium:

The medium of instruction shall be English.

Passing minimum:

Passing Eligibility and Classification for the award of the Degree are as per the norms of the Choice Based Credit System.

Conditions for Affiliation:

The following are the requirements for the grant of affiliation for M.Sc. Mathematics programme in the institutions affiliated to Pondicherry University:

- (i) The institution shall have conducted the B.Sc. Mathematics programme for a minimum period of 6 years
- (ii) Faculty strength for M.Sc. Mathematics programme: 3 regular Faculty in the first year.
Additional 3 regular faculty in the second year.
A total of 6 regular faculty for the whole programme.

- (iii) Qualifications for the faculty: The faculty shall possess the qualifications as prescribed by UGC.
- (iv) Recruitment of faculty: The recruitment of faculty shall be through a duly constituted Selection Committee with a nominee of the University, by advertisement
- (v) Class rooms: 2 permanent rooms with furniture, platform and black board.
- (vi) Faculty room: 1 permanent room with furniture.
- (vii) Computers: 3 for the faculty.
- (viii) Library:
 - Books: 10 copies of each prescribed text book;
 - 1 copy of each prescribed reference book;
 - 3 reference books for each hard core subject.
 - Journals: Minimum 2
 - Library Room: 1 with furniture for books
 - Reading Space for Students
- (ix) Computer Lab: required if computer papers are offered as soft core with 1 computer for every 2 students

SYLLABUS

1. This syllabus is effective for the candidates admitted from the Academic year 2009 – 2010 onwards.
2. The syllabus will be common for both the University Department and the affiliated institutions.
3. Choice Based Credit System (CBCS) will be followed for the University Department. Non-CBCS system will be followed for the affiliated institutions.
4. During the four semesters, a candidate has to earn a minimum of **72 credits by studying 12 Hard Core Courses and sufficient number of Soft Core Courses.**
5. Under CBCS system, each course will have **4 credits.**

List of Hard Core Courses

S.NO	COURSE CODE	COURSE TITLE
1	MATH-411	Advanced Algebra
2	MATH-412	Linear Algebra
3	MATH-413	Real Analysis
4	MATH-414	Topology
5	MATH-421	Differential Geometry
6	MATH-422	Lebesgue Measure Theory
7	MATH-424	Ordinary Differential Equations
8	MATH-425	Discrete Mathematics
9	MATH-426	Introduction to Complex Analysis
10	MATH-511	Fluid Mechanics
11	MATH-512	Analytical Dynamics
12	MATH-513	Functional Analysis

List of Soft Core Courses

S.NO	COURSE CODE	COURSE TITLE
1	MATH-522	Graph Theory with Applications
2	MATH-523	Graph Theory with Algorithms
3	MATH-524	Number Theory
4	MATH-525	Operations Research
5	MATH-527	Algebraic Number Theory
6	MATH-528	Advanced Algebraic Number Theory
7	MATH-529	Theory of Fuzzy Sets
8	MATH-530	Algebraic Coding Theory
9	MATH-531	Cryptography
10	MATH-532	Automata Theory
11	MATH-533	Advanced Topics in Topology and Analysis
12	MATH-534	Approximation Theory
13	MATH-535	Advanced Functional Analysis
14	MATH-536	Difference Equations
15	MATH-537	Partial Differential Equations
16	MATH-538	Lie Groups of Transformations and Differential Equations
17	MATH-539	Numerical Analysis for Ordinary Differential Equations
18	MATH-540	Advanced Fluid Mechanics
19	MATH-541	Integral Equations
20	MATH-542	Advanced Mathematical Analysis
21	MATH-543	Representation Theory of Compact Groups
22	MATH-544	Elements of Harmonic Analysis
23	MATH-545	Linear Lie Groups
24	MATH-546	Graph Theory
25	MATH-548	Advanced Topics in Discrete Mathematics
26	MATH-549	Laboratory Practical in Mathematics
27	MATH-550	Topics in Topology and Analysis
28	MATH-551	Functional Analysis – II
29	MATH-552	Operator Theory
30	MATH-554	Non-commutative Rings and Representations
31	MATH-555	Complex Analysis
32	MATH-556	Algorithms using C++
33	MATH-557	Algorithms using Java

PONDICHERRY UNIVERSITY
DEPARTMENT OF MATHEMATICS

SEMESTER WISE COURSE LIST

S.NO	COURSE CODE	TITLE OF THE COURSE	NATURE OF THE COURSE	NO. OF CREDITS
I SEM -----				
1.	MATH-411	Advanced Algebra	Hard Core	4
2.	MATH-412	Linear Algebra	Hard Core	4
3.	MATH-413	Real Analysis	Hard Core	4
4.	MATH-414	Topology	Hard Core	4
5.		Elective – I	Soft Core	4
II SEM -----				
1.	MATH-421	Differential Geometry	Hard Core	4
2.	MATH-422	Lebesgue Measure Theory	Hard Core	4
3.	MATH-424	Ordinary Differential Equations	Hard Core	4
4.	MATH-425	Discrete Mathematics	Hard Core	4
5.	MATH-426	Introduction to Complex Analysis	Hard Core	4
III SEM -----				
1.	MATH-511	Fluid Mechanics	Hard Core	4
2.	MATH-512	Analytical Dynamics	Hard Core	4
3.	MATH-513	Functional Analysis	Hard Core	4
4.	MATH-524	Number theory	Soft Core	4
5.	MATH-539	Numerical Analysis for Ordinary Differential Equations	Soft Core	4
6.	MATH-542	Advanced Mathematical Analysis	Soft Core	4
IV SEM -----				
1.		Elective - II	Soft Core	4
2.		Elective - III	Soft Core	4
3.		Elective – IV	Soft Core	4
4.		Elective – V	Soft Core	4
5.		Elective – VI	Soft Core	4

M.Sc. MATHEMATICS PROGRAMME IN AFFILIATED INSTITUTIONS

COURSE PATTERN

FIRST SEMESTER:

Advanced Algebra

Real Analysis

Discrete Mathematics

Optional: Difference Equations

SECOND SEMESTER:

Linear Algebra

Introduction to Complex Analysis

Analytical Dynamics

Optional: Graph Theory

Optional: Algorithms using C++

THIRD SEMESTER:

Topology

Lebesgue Measure Theory

Fluid Mechanics

Optional: Complex Analysis

Optional: Operations Research

FOURTH SEMESTER:

Functional Analysis

Differential Geometry

Ordinary Differential Equations

Optional: Graph Theory with Applications

SYLLABUS

HARD CORE : MATH-411 ADVANCED ALGEBRA

1. The class equation of a finite groups - Sylow theorems - Direct products.
2. Polynomial rings- Polynomials over the rational field – Polynomial rings over Commutative rings.
3. Field extensions - Algebraic and transcendental extensions, Separable and inseparable extensions – Normal extensions.
4. Perfect Fields, Primitive elements, Algebraically closed fields, Galois extensions, Fundamental theorem of Galois theory.
5. Solvable groups - Solvability by radicals and insolvability of the general equation of degree 5 - Finite fields.

Text book

Treatment and Contents as in I.N.Herstein: Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.

Sections – 2.11, 2.12 and 2.13

Sections – 3.9, 3.10 and 3.11

Sections – 5.1, 5.3 and 5.5

Sections - 5.6

Sections – 5.7 and 7

Reference books

1. M.Artin: Algebra, Prentice-Hall of India, 1991.
2. N.Jacobson: Basic Algebra, Volumes I & II, W.H.Freeman, 1980.
3. S.Lang: Algebra, 3rd edition, Addison-Wesley, 1993.
4. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul: Basic Abstract Algebra (2nd Edition) Cambridge University Press, Indian edition, 1997.

HARD CORE : MATH-412 LINEAR ALGEBRA

1. The Algebra of Linear transformations, characteristic roots, Similarity of linear transformations, Invariant subspaces and Matrices.
2. Reduction to triangular forms, Nilpotent transformations, Index of nilpotency and Invariant of Nilpotent transformation.
3. Jordan blocks and Jordan forms, Modules - Cyclic modules - Fundamental theorem on modules over PID.
4. Rational canonical form, trace, transpose and Determinants.
5. Hermitian, Unitary and Normal transformations - Real quadratic forms.

Text book

Treatment and Contents as in I.N.Herstein: Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.

Sections – 6.1, 6.2 and 6.3

Sections – 6.4 and 6.5

Sections – 6.6 and 4.5

Sections - 6.7,6.8 and 6.9

Sections – 6.10 and 6.11

Reference books

1. M.Artin: Algebra, Prentice-Hall of India, 1991.
2. N.Jacobson: Basic Algebra, Volumes I & II, W.H.Freeman, 1980.
3. S.Lang: Algebra, 3rd edition, Addison-Wesley, 1993.
4. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul: Basic Abstract Algebra (2nd Edition) Cambridge University Press, Indian edition, 1997.

HARD CORE: MATH-413 REAL ANALYSIS

1. Riemann Stieltjes Integral - Properties – Integration and Differentiation – Rectifiable Curves.
2. Sequence and Series of Functions – Uniform Convergence of Sequence of Functions – Cauchy’s Criterion for Uniform convergence – Weierstrass M-Test – Abel’s and Dirichlet’s Test for Uniform Convergence – Uniform Convergence and Continuity – Weierstrass Approximation Theorem.
3. Improper Integrals – Improper Integrals of Bounded Functions – Infinite Integrals – Cauchy Criterion – Comparison – Dirichlet Tests – Absolute Convergence – Uniform Convergence and Infinite Integrals - Infinite Integrals depending on a parameter.
4. Differentiability of Functions From an Open subset of \mathbb{R}^n to \mathbb{R}^m – Properties – Chain Rule – Partial and directional Derivatives. Continuously Differentiable Functions – Contraction mapping Theorem – Inverse Function Theorem.
5. Implicit Function Theorem – Interchange of order of Differentiation – Taylor’s Series – Extrema of a Function – Extremum Problem with Constraints –Lagrange Multiplier Method (Statement only) –Applications.

Text books

For Units I, II, IV and V W.Rudin: Principles of Mathematical Analysis, Mc Graw Hill, 1976.

For Units II, R.G.Bartle: The Elements of Real Analysis, Wiley International Edition, 1976.

HARD CORE : MATH-414 TOPOLOGY

1. Sets – Functions – Products of Sets – Relations - Countable sets – Uncountable sets – Metric spaces (Basic Definitions) – Convergence of a sequence – Completeness for metric spaces – Baire's theorem.
2. Topological spaces – Elementary concepts – Open bases - Open sub bases – Weak topologies.
3. Compact spaces – Products of spaces – Tychonoff's theorem - Locally compact spaces – Compactness for metric spaces.
4. T_1 spaces - Hausdorff spaces - Completely regular spaces - Normal spaces – Urysohn's lemma - Tietze extension theorem.
5. Connected spaces – Components of a space – Totally disconnected spaces – Locally connected spaces.

Text book

Treatment as in G.F.Simmons: An Introduction to Topology and Modern Analysis.

Chapter 1 – Sections 1 to 7

Chapter 2 – Sections 9 to 12

Chapter 3 – Sections 16 to 19

Chapter 4 – Sections 21 to 24

Chapter 5 – Sections 26 to 28

Chapter 6 – Sections 31 to 34

Reference books

1. J.R.Munkres: Topology, Pearson Education Inc., Second Edition, 2000.
2. J.Dugundgi: Topology, Allyn and Bacon, Boston, 1966.

HARD CORE : MATH-421 DIFFERENTIAL GEOMETRY

1. Graphs and level sets - Vector fields - The tangent space
2. Surfaces - Vector fields on surfaces - Orientation - The Gauss map
3. Geodesics - Parallel transport
4. The Weingarten map - Curvature of plane curves
5. Arc length and line integrals - Curvature of surfaces

Text book

Treatment as in J.A. Thorpe: Elementary Topics in Differential Geometry, Springer, 2004.

Chapters 1 to 12.

HARD CORE : MATH-422 LEBESGUE MEASURE THEORY

1. Ring and algebra of sets- Sigma algebras- Examples- Algebras and Sigma algebras generated by a class of sets - Borel algebra and Borel sets. Lebesgue outer measure on \mathbb{R} - Countable sub-additivity - Measurable sets –

Examples - Sigma algebra structure of measurable sets - Countable additivity of Lebesgue measure on \mathbb{R} - Cantor set.

2. Construction of a non- measurable subset of $[0, 1]$ - Measurable functions- Examples and basic properties - Approximation of measurable and bounded measurable functions by simple measurable functions - Approximation by step functions and continuous functions - Egorov's theorem.

3. Lusin's theorem - Lebesgue integral of non- negative measurable functions- Integrable functions and Lebesgue integral of integrable functions - Linearity- Monotone convergence theorem - Fatou's lemma - Dominated convergence theorem- Applications of convergence theorems.

4. Comparison of Riemann and Lebesgue integration - Lebesgue integrability of Riemann integrable functions - Characterization of Riemann integrable functions – Improper Riemann integrals and their Lebesgue integrals - Riemann- Lebesgue lemma - Functions of bounded variation - Statement of Vitali's lemma and theorem on almost everywhere differentiability of monotone increasing functions.

5. Absolutely continuous functions - Examples and properties - Absolute continuity of indefinite integral of Lebesgue integrable functions - Differentiation of indefinite integrals - Characterization of absolutely continuous functions as indefinite integrals.

Text books

1. Royden: Real Analysis (Units 1, 2, 4 and 5).
2. Walter Rudin: Real and Complex Analysis (Unit 3).

Reference book

P.R. Halmos: Measure Theory.

HARD CORE : MATH-424 ORDINARY DIFFERENTIAL EQUATIONS

1. Qualitative properties of Solutions – The Sturm Comparison Theorem – Eigen values and Eigen functions and Vibrating String.
2. Series Solutions of First Order Equations – Second Order Linear Equations – Ordinary points - Regular Singular Points – Gauss Hyper Geometric Equations.
3. Legendre Polynomials – Properties of Legendre Polynomials – Bessel Functions- The Gamma Function - Properties of Bessel Function.
4. Linear Systems – Homogeneous Linear System with Constant Coefficients.
5. The Existence and Uniqueness of Solutions – The method of Successive Approximations – Picards's Theorem.

Text book

Treatment as in G.F.Simmons: Differential Equations with Applications and Historical Notes, McGraw Hill Book Company, 1972.

Sections 22-30, 32-35, 37-38 and 55-56.

HARD CORE: MATH 425 – DISCRETE MATHEMATICS

I. Inclusion (equality of sets) – Power Set – Cartesian (Products) – Relations – Equivalence Relations - Partial Ordering – Partially Ordered Set (Representation and associated terminology) – Lattices as Partially Ordered Sets– Properties of Lattices - Lattices as Algebraic Systems-Sublattices –Direct Product – Homomorphism.

II. Special Lattices (Complete lattices, bounded lattices, Complemented lattices, distributive lattices and their properties)-Boolean Algebra – Sub-algebra – Direct Product - Homomorphism

III. Stone’s Theorem (join-irreducible elements, atoms) – Boolean Forms (minterms, sum-of-products canonical form) – Free Boolean Algebra – values of Boolean Expressions (a binary valuation process) – Boolean Functions – Symmetric Boolean Expressions.

IV Graphs and simple Graphs – Graph Isomorphism – Complete Graphs – Bipartite Graphs – Complements and self Complementary Graphs – Incidence And Adjacency Matrices – Subgraphs – Vertex Degrees – Degree – Graphic Sequences – Paths – Connection – Components – Cycles and Characterization of Bipartite Graphs.

V. Trees – Cut edges and Bonds – Cut Vertices – Cayley’s Formula – Connectivity – Blocks – Euler Tours – Hamilton cycles.

Text Books:

1. Trembley, J.P and Manohar, R : Discrete Mathematics Structures with Applications to Computer Science, Mc Graw Hill Book Company, 1997.
Chap – 2: Sec . 2-1.1, 2-1.2, 2-1.3, 2-1.4, 2-1.6, 2-1.8, 2-1.9, 2-3.1, 2-3.2, 2-3.5, 2-3.8, 2-3.9.
Chap-4: Sec 4-1, 4-2, 4-3.
2. Bondy J. A. and Murthy U.S.R., Graph Theory with Applications. Mac Comp.
Chap 1: Sec. (1.1) to (1.7)
Chap 2: Sec. (2.1) to (2.4)
Chap 3: Sec. (3.1) to (3.2)
Chap 4: Sec. (4.1) to (4.2)

Reference Books:

1. Grimaldi R.P. Discrete and Combinatorial Mathematics: An Applied Introduction, Pearson Edn Asia. Delhi – 2002.
2. Kolman B. Busby R. C, and Ross S.C., Discrete Mathematical Structures, Pearson Edn Pvt. Ltd, New Delhi – 2003.
3. Rosen K., Discrete Mathematics and its Applications, Tata Mc Graw Hill Pub. Com. Ltd, New Delhi – 2003.
4. Liu C. L., Elements of Discrete Mathematics, Mc Graw Hill Book Company.
5. Seymour Lipschutz, Finite Mathematics, Mc Graw Hill Book Com (International Edition), New York, 1983.
6. Wiitala: Discrete Mathematics, A unified Approach, Mc Graw Hill Book Company.
7. Harary F. Graph Theory, Addison – Wesley Reading Mass (1969).
8. Wilson R. J., Introduction to Graph Theory, Oliver and Boyd, Edinburgh (1972).

HARD CORE : MATH-426 INTRODUCTION TO COMPLEX ANALYSIS

1. Analytic Functions

Power series - Analytic functions - exponential function - Logarithm function - Cauchy-Riemann equations - Analytic functions as mappings, Mobius transformations - Power series representation of analytic functions.

2. Complex Integration

Zeros of an analytic function - Liouville's theorem - Fundamental theorem of Algebra - Maximum modulus theorem - The index of a closed curve - Cauchy's theorem and integral formula - Morera's theorem - The homotopic version of Cauchy's theorem and simple connectivity.

3. Singularities

Counting zeros - the open mapping theorem - Goursat's theorem - Classification of singularities - Laurent series development - Residue theorem.

4. The Maximum Modulus Theorem

The argument principle - Rouché's theorem - The maximum principle - Schwarz's Lemma.

5. Harmonic Functions

Basic properties of harmonic functions - Mean value theorem - Harmonic functions on a disk - Harnack's inequality - Harnack's theorem.

Text book

John B. Conway: Functions of One Complex Variable, Second edition, 1980

HARD CORE : MATH-511 FLUID MECHANICS

1. Equations of motion - Euler's Equation – Conservation of Mass – Balance of Momentum – Transport Theorem - Conservation of Energy – Incompressible Flows – Isentropic Fluids – Bernoulli's Theorem.
2. Rotations and Vorticity – Kelvin's Circulation Theorem – Helmholtz's Theorem.
3. Navier- Stokes Equations – Scaling Properties – Decomposition Theorem - Stokes Equations – Poiseuille Flow .
4. Potential Flow – Complex Potential – Blasius Theorem - Kutta-Joukowski Theorem – D'Alembert's Paradox – Stokes Paradox.
5. Boundary Layers – Prandtl Boundary Layer Equations – Steady Boundary Layer Flow on a Flat Plate of Infinite Width.

Text book

A.J.Chorin and J.E Marsden: 'A Mathematical Introduction to Fluid Mechanics,' Texts in Applied Mathematics 4, Springer Verlag, 1990.

HARD CORE : MATH-512 ANALYTICAL DYNAMICS

Unit I

Generalized coordinates-Virtual displacements-D'Alembert's principle and derivation of the Lagrange equations.

Unit II

Lagrange equation for non-holonomic constraints-Method of Lagrange multipliers-Velocity dependent potentials-Non-conservative forces and dissipation function- Non-holonomic systems and Lagrange multipliers.

Unit III

Hamilton's equation-The Hamilton principle-Variational principle

Unit IV

Canonical transformations-Hamilton-Jacobi theory.

Unit V

Dissipative systems-Attractors-Equilibrium solutions-Limit cycles-Flotquet's theory of stability.

Text book

Walter Greiner, Classical Mechanics: Systems of Particles and Hamiltonian Dynamics, Springer, ISE, 2004.

Unit I: Sections 14-15; Unit II: Sections 16-17; Unit III: Section 18 (341-364); Unit IV: Sections 19-20; Unit V: Sections 21-22 (419-451)

Reference books

1. H. Goldstein, Classical Mechanics, Narosa Publishing House, New Delhi, 1985.
2. F. Scheck, Mechanics: From Newton's Laws to Deterministic Chaos, Springer, 1999.

HARD CORE : MATH-513 FUNCTIONAL ANALYSIS

1. Normed linear spaces – Examples – Spaces c_0 , l_p and $L_p[a, b]$, $1 \leq p \leq \infty$ - Linear transformations – Examples – Continuity and boundedness – Linear functionals – Continuity – Examples of discontinuous linear transformations – Dual spaces – Completeness of sequence spaces c_0 and l_p , $1 \leq p \leq \infty$ - Completeness of $B(X, Y)$, the space of bounded, linear transformations from X into Y and space $C(Q)$, Q compact and Hausdorff, with sup norm – Completeness of finite dimensional normed linear spaces.
2. Compactness of unit ball of finite dimensional spaces – Riesz's lemma – Equivalence of norms and continuity of linear transformations for finite dimensional spaces – Separability of c_0 , l_p , $1 \leq p \leq \infty$ and $C[a, b]$ – Non-separability of l_∞ - Duals of c_0 and l_p , $1 \leq p \leq \infty$ spaces.
3. Hahn-Banach theorem for real vector spaces – Hahn-Banach theorem for real and complex normed linear spaces – Standard corollaries – Normed linear spaces with separable duals are separable – The Baire's category theorem – The Principle of Uniform boundedness – Applications.
4. Closed and open maps – Maps with closed graph – Example of discontinuous, linear map with closed graph – Open mapping theorem and the closed graph theorem – Applications – Natural embedding into second dual – Reflexive spaces – Inner product spaces – Examples – Inner product spaces and parallelogram law for norm – Orthonormal sets – Bessel's inequality – Gram-Schmidt orthonormalization – Orthonormal basis.
5. Separable Hilbert spaces and countable orthonormal basis – Linear isometry onto l_2 - Example of a non-separable Hilbert space – Uncountable orthonormal basis and definition of convergence of Fourier series – Riesz-Fisher's theorem. Orthogonal projections – Closed subspaces are Chebychev - Riesz's representation theorem – Reflexivity of Hilbert spaces.

Text book

M. Thamban Nair: Functional Analysis.

Reference book

B.V.Limaye: Functional Analysis.

SOFT CORE : MATH-522 GRAPH THEORY WITH APPLICATIONS

Treatment as in J.A. Bondy and U.S.R. Murthy: Graph Theory with Applications, 1976.

Pre requisites: Graphs and simple graphs - Special graphs (Complete graphs, Complement of graphs and null graphs) - Graph isomorphism – Sub graphs - Vertex degrees - Degree sequences and graphic sequences - walks, paths, Cycles - Graph connection and components - Bipartite graphs and their characterizations.

(Chapter 1. No questions from this chapter)

1. Connectivity and edge connectivity - Vertex cuts and edge cuts - Whitney's inequality (relating K, K and d) - Blocks and blocks of graphs - Characterization of 2 - connected graphs and blocks - Menger's theorem (without proof).
(Chapter 3 in which Section (3.3) is omitted).
2. Independent sets and their characterization - Matchings - Vertex as well as edge independence numbers, Covering numbers - Perfect matching – Konig's Theorem (with out proof) - Galli's theorem - Ramsey numbers - Theorems on the upper bounds and lower bounds for Ramsey numbers - Ramsey graphs - Erdos theorem.
(Chapter 7 - Sections (7.1) and (7.2) only).
3. Vertex colourings and chromatic numbers of graphs - Critical graphs and their properties - Brook's theorem - Hajo's conjecture and Dirac's theorem.
(Chapter 8 - Sections (8.1), (8.2) and (8.3) only)
4. Chromatic polynomials - The five colour theorem - The four colour theorem (without proof) - Edge chromatic number - Vizing's theorem (statement only).
(Chapter 8 - Section (8.4) only; Chapter 9 - Section (9.6) only; Chapter 6 - Sections (6.1) and (6.2) only)
5. Directed graphs - Directed paths (Roy-Gallai Theorem) - Tournaments - Directed Hamilton paths and cycles (Moon's theorem, Ghouila - Hourri theorem).
(Chapter 10 - Sections (10.2) and (10.3) only)

Reference books

1. F. Harary: Graph Theory, 1969.
2. Narasinga Deo: Graph Theory with Applications to Engineering and Computer Science, Prentice Hall of India Private Limited, New Delhi.
3. R.J. Wilson: Introduction to Graph Theory, 1972.
4. L.R. Foulds: Graph Theory Applications, 1993.
5. S.A. Choudam: A First Course in Graph Theory, 1987.

SOFT CORE : MATH-523 GRAPH THEORY WITH ALGORITHMS

1. Graphs – degrees – Isomorphism – subgraphs – Walks, paths, Circuits - connected graphs – components – Euler graphs – Operations on graphs – Hamiltonian paths and circuits – Travelling Salesman Problem

(Chapters 1 and 2 in which Sections (1.2),(1.6) and (2.3) are omitted).

2. Trees – some properties of trees – pendant vertices in a tree – distance and centers in a tree – Rooted and binary trees – On counting trees – Spanning trees – Fundamental circuits – finding all spanning trees of graph – spanning trees in a weighted graph.

(Chapter 3).

3. Cut sets – some properties of a cutset – All cut sets in a graph – Fundamental circuits and cut sets – connectivity and separability – Network flows – (1) Isomorphism – (2) Isomorphism.

(Chapter 4)

4. Incidence matrix – Sub matrices - Circuit matrix – Fundamental Circuit matrix and rank – An application to a switching network – Cut set matrix – Relationships – path matrix .

(Chapter 7).

5. Algorithms – (Input) Computer representation of a graph – the out pat – some basic graph theoretic algorithms: connectedness and components, A spanning tree – A set of fundamental circuits, cut vertices and separability – shortest path algorithms: Shortest path either from a specified vertex to another specified vertex or among all pairs of vertices.

(Chapter 11 in which Sections (11.4) of Algorithm 5, (11.6) of Algorithm 8, (11.7), (11.8), (11.9) and (11.10) are omitted).

Text book

Narsingh Deo: Graph Theory with Applications to Engineering and Computer Science, Prentice Hall of India Private Limited, New Delhi.

Reference books

1. F. Harary: Graph Theory, Addison Wesley, 1969.
2. C.L.Liu: Elements of Discrete Mathematics, McGraw Hill Book Company, Second Edition, 1986.
3. R. Johnsonbaugh: Discrete Mathematics, 1989.
4. L.R. Foulds: Graph Theory Applications, Narosa Publishing House, 1993.

SOFT CORE : MATH-524 NUMBER THEORY

1. Divisibility : Introduction - Divisibility - Primes.
2. Congruences - Solution of Congruences - Congruences of higher degree - Prime power moduli - Prime Modulus - Congruences of degree two, prime modulus - Power residues - Number theory from an algebraic view point
Multiplicative groups, rings and fields.
3. Quadratic Reciprocity: Quadratic Residues - Quadratic Reciprocity - The Jacobi Symbol.
4. Some functions of Number Theory: Greatest Integer function - Arithmetic function s - The Moebius Inversion Formula - Multiplication of arithmetic functions – Recurrence functions.
5. Some Diophantine Equations: The equation $ax+by = c$ - Positive Solutions - Other linear equations - The equation $x^2+y^2 = z^2$ - The Equation $x^4+y^4 = z^2$ - Sum of fourth powers - Sum of two squares - The equation $4x^2 + y^2 = n$.

Text book

Treatment as in Ivan Niven and S.Zuckerman: An Introduction to the Theory of Numbers, John Wiley, New York, 2000.

Chapter 1: Sections 1.1 – 1.3

Chapter 2: Sections 2.1 - 2.11

Chapter 3: Sections 3.1 – 3.3

Chapter 4: Sections 4.1 – 4.5

Chapter 5: Sections 5.1 - 5.6, 5.10 and 5.11

SOFT CORE : MATH-525 OPERATIONS RESEARCH

1. Hyperplanes and half-spaces – Supporting and separating hyper planes – Convex functions – Linear programming basic concepts – Convex sets – Linear programming problems – Examples of LPP – Feasible, basic feasible and optimal solutions – Extreme points.
2. Linear Programming – Graphical Method - Simplex Method.
3. Network models – Network definitions – Minimal spanning tree algorithm – Shortest route problem.
4. Integer Programming – Cutting plane algorithm - Branch and Bound Technique.
5. Decisions under risk – Decision trees – Decision under uncertainty.
Game Theory - Two - Person, Zero - Sum Games - Games with Mixed Strategies - Graphical Solution - Solution by Linear Programming.

Text books

1. Hamdy A. Taha: Operations Research, Fourth Edition, Chapter 8 – Sections 8.3, 8.4 and Chapter 11– Sections 11.1 to 11.4.
2. J.K.Sharma: Mathematical Models in Operations research, Chapter 2 – Sections 2.12 to 2.14 and Chapter 4 – Sections 4.3 to 4.4.

SOFT CORE : MATH-527 ALGEBRAIC NUMBER THEORY

1. Elementary Number Theory

Integers – Greatest common divisor – Infinitude of primes – Unique factorization in \mathbb{Z} – Fermat's little theorem – Euler's Φ function and Euler's theorem – Multiplicative property of Φ function – Applications of unique factorization – The equation $x^2 + y^2 = z^2$ – The equation $x^4 + y^4 = z^2$ – The equation $x^4 - y^4 = z^2$ – Fermat numbers and their properties.

2. Euclidean Rings

Preliminaries: Units, Associates, Irreducible elements, Norm map, Unique factorization domain, Principal ideal domain, Euclidean domain – Gauss' lemma – Gaussian integers – Units and primes in the ring of Gaussian integers – Eisenstein integers – Units in the ring of Eisenstein integers – Factorization of 3 – Order of $\mathbb{Z}[\rho] / (\lambda)$.

3. Algebraic Numbers and Integers

Basic concepts – Algebraic number – Algebraic integer – Minimal polynomial – Countability of algebraic numbers – Liouville's theorem for \mathbb{R} – Algebraic number fields – Theorem of the primitive element – Liouville's theorem for \mathbb{C} – Characterization of algebraic integers.

4. Integral Bases

The norm and the trace – Integral basis for an algebraic number field – Algebraic integers of $\mathbb{Q}(\sqrt{-5})$ – Existence of an integral basis – Discriminant of an algebraic number field – Index – Determination of an integral basis for the ring of integers of a quadratic number field.

5. Dedekind Domains

Integral closure – Integrally closed ring – Noetherian ring – Dedekind domain – Characterizing Dedekind domains.

Text book

J.Esmonde and M.Ram Murty: Problems in Algebraic Number Theory, Graduate Texts in Mathematics, Volume 190, Springer Verlag, New York, 1999.

Sections 1.1 and 1.2

Sections 2.1, 2.2 and 2.3

Sections 3.1, 3.2 and 3.3

Sections 4.1, 4.2 and 4.3

Sections 5.1 and 5.2

SOFT CORE : MATH-528 ADVANCED ALGEBRAIC NUMBER THEORY

1. The Ideal Class Group

Euclidean rings – Hurwitz constant – Fractional ideals – Finiteness of the ideal class group – The class number of an algebraic number field – The class number of $\mathbb{Q}(\sqrt{-5})$ – The Diophantine equation $x^2 + 5 = y^3$.

2. Quadratic Reciprocity

Preliminaries – Quadratic residues and quadratic non residues – The Legendre symbol – The quadratic character of -1 and 2 – Gauss sums – The law of quadratic reciprocity.

3. The Structure of Units

Discrete subgroup of \mathbb{R}^m – Dirichlet's unit theorem – Units in real quadratic fields – Pell's equation.

4. Higher Reciprocity Laws

Cubic reciprocity – Eisenstein reciprocity.

5. Analytic Methods

The Riemann and Dedekind zeta functions – Zeta functions of quadratic fields – Dirichlet's hyperbola method.

Text book

J.Esmonde and M.Ram Murty; Problems in Algebraic Number Theory, Graduate Texts in Mathematics, Volume 190, Springer Verlag, New York, 1999.

Sections 6.1, 6.2 and 6.3

Sections 7.1, 7.2 and 7.3

Sections 8.1 and 8.2

Sections 9.1 and 9.2

Sections 10.1 and 10.2

SOFT CORE : MATH-529 THEORY OF FUZZY SETS

1. Fuzzy sets

Basic definitions – Types of fuzzy sets – Basic concepts – α cuts and their properties – Representations of fuzzy sets – first and second decomposition theorems.

2. Operations on fuzzy sets

Types of operations – Fuzzy complements – Fuzzy intersections: t norms – Fuzzy unions: t conorms – Combinations of operations.

3. Elements of fuzzy arithmetic

Fuzzy numbers – Linguistic variables – Arithmetic operations on intervals – Arithmetic operations on fuzzy numbers – Fuzzy equations.

4. Fuzzy relations

Crisp and fuzzy relations – Projections and cylindric extensions – Binary fuzzy relations – Binary relations on a single set – Fuzzy equivalence relations – Fuzzy compatibility relations.

5. Fuzzy logic

An overview of classical logic – multi valued logics – Fuzzy propositions – Fuzzy quantifiers – Linguistic hedges – Inference from conditional fuzzy propositions.

Text book

George J. Klir and Bo Yuan: Fuzzy sets and Fuzzy Logic: Theory and Applications, Prentice – Hall of India Private Limited, New Delhi, 2000.

Reference book

H.J. Zimmermann: Fuzzy set theory and its Applications, Allied Publishers Limited, New Delhi, 1991.

SOFT CORE : MATH-530 ALGEBRAIC CODING THEORY

1. Error detection, Correction and decoding

Communication channels – Maximum likelihood decoding – Hamming distance – Nearest neighbourhood minimum distance decoding – Distance of a code.

2. Linear codes

Linear codes – Self orthogonal codes – Self dual codes – Bases for linear codes – Generator matrix and parity check matrix – Encoding with a linear code – Decoding of linear codes – Syndrome decoding.

3. Bounds in coding theory

Sphere covering bound – Gilbert Varshamov bound – Binary Hamming codes – q-ary Hamming codes – Golay codes – Singleton bound and MDS codes – Plotkin bound.

4. Cyclic codes

Definitions – Generator polynomials – Generator matrix and parity check matrix – Decoding of Cyclic codes.

5. Special cyclic codes

BCH codes – Parameters of BCH codes – Decoding of BCH codes – Reed Solomon codes.

Text book

San Ling and Chaoping Xing; Coding Theory: A first course, Cambridge University Press, 2004.

Reference book

V. Pless: Introduction to the Theory of Error correcting codes, Wiley, New York, 1982

SOFT CORE : MATH-531 CRYPTOGRAPHY

1. Basic concepts

Factoring and primality testing – Perfect numbers – Fermat's divisibility test – Fermat numbers – Base representation of integers – Computational complexity.

2. Symmetric key crypto systems

An overview of congruences – Block ciphers – The DES key Schedule – The DES Cryptosystem

3. Public key cryptosystems

Exponentiation, discrete logs and protocols – Public key cryptography – RSA system – Rabin system – Elgamal system.

4. Authentication and knapsack

Digital signatures – Signature schemes related to public key Crypto Systems – Knapsack problem – Merkle Hellman system – Chor Rivest system.

5. Primality testing

Primitive roots – Gauss's algorithm – Primitive root theorem – Index calculus – Mersenne number – Pocklington's theorem – Proth's theorem – Pepin's primality test.

Text book

Richard A. Mollin: An Introduction to Cryptography, Chapman & Hall / CRC, Boca Raton, 2000.

Reference book

Dominic Walsh: Codes and Cryptography, Oxford Science Publications, Clarendon Press, Oxford, 1988.

SOFT CORE : MATH-532 AUTOMATA THEORY

1. Introduction to the theory of computation

Three basic concepts: Languages, Grammars, Automata – Some application.

2. Finite Automata

Deterministic finite accepters – Nondeterministic finite accepters – Equivalences of deterministic and nondeterministic finite accepters – Reduction of the number of states in finite automata.

3. Regular Languages and Regular Grammars

Regular expression – Connection between regular expression and regular languages - Regular grammars.

4. Properties of Regular Languages

Closure properties of regular languages – Elementary questions about regular languages – Identifying non regular languages.

5. Context-Free Languages

Context-free grammars – Parsing and ambiguity – Context – Free Grammars and programming languages.

Text book

Treatment as in Peter Linz: “An Introduction to Formal Languages and Automata”.

SOFT CORE : MATH-533 ADVANCED TOPICS IN TOPOLOGY AND ANALYSIS

1. Quotient topology and quotient maps - Examples of quotient spaces - Path connectedness - Standard results - Example of a connected but not path connected space- Locally connected spaces.
2. The Uryshon's metrization theorem - One point compactification - Stone- Cech compactification - The Arzela - Ascoli theorem.
3. Local finiteness- Countably locally finite refinement of open coverings of metric spaces – Paracompactness - Standard results - Metric spaces are paracompact.
4. Partition of unity - L_p - spaces – Completeness - Dual of $L_p[a, b]$ for $1 \leq p < \infty$.
5. Extreme points - Caratheodory's theorem - Krein- Milman theorem - Milman converse theorem - Extreme points of the closed unit ball of $c, l_\infty, C(Q), Q$ compact, Hausdorff and the dual of $C(Q)$ - Strictly convex spaces - Examples.

Reference books

1. James R. Munkres: Topology
2. James Dugundji: General Topology
3. Joseph Conway: A Course on Functional Analysis
4. M. Fabian, P.Habala, P. Hajek, V.M. Santalucia, J.Pelant and V. Zizler: Functional Analysis and Infinite Dimensional Geometry, CMS Books in Mathematics, Springer-Verlag, 2001.

SOFT CORE : MATH-534 APPROXIMATION THEORY

1. Interpolation by polynomials - Lagrange interpolation - Vander Monde's determinant- Bernstein polynomials - Weierstrass approximation theorem.
2. Stone- Weierstrass theorem (Real and complex versions) - Weierstrass theorem as corollary - Approximation of continuous, periodic functions by trigonometric polynomials - Best approximation in $C[a, b]$ with sup norm - Chebychev's Alternation theorem - Theorem of de La Vallee Poussin.
3. General linear families - Haar system and its characterizations - Uniqueness of polynomials of best approximation - Strong unicity theorem - Harr's unicity theorem.
4. An algorithm of Remes and convergence under Haar condition - Strictly convex and uniformly convex Banach spaces - Approximation in inner product spaces – Approximation from closed, convex subsets - Approximation from subspaces of Hilbert spaces - Uniform convexity and continuity of metric projection.
5. Approximation from finite dimensional subspaces - Normal equations and Gram's determinant - approximation in $L^2[a, b]$ - Orthogonal polynomials - Legendre and Chebychev polynomials.
Best approximation by subspaces of Banach spaces - Duality formula - Spaces in which all closed subspaces are proximal or Chebychev-proximality of weak* closed subspaces - Approximation by closed hyperplanes.

Reference books

1. E.W.Cheney: An Introduction to Approximation theory.
2. B.V. Limaye: Functional Analysis.
3. Frank Deutsch: Best approximation in inner product spaces.
4. Serge Lang: Real Analysis.
5. Ivan Singer: Best approximation in normed linear spaces by elements of linear subspaces.

SOFT CORE : MATH-535 ADVANCED FUNCTIONAL ANALYSIS

1. Topological vector spaces-balanced and absorbing sets - Locally convex spaces – Examples- Weak and weak* topologies.
2. Finite dimensional topological vector spaces -Minkowski functional - bounded and totally bounded sets - Metrizable topological vector spaces - Characterization of normable locally convex spaces.
3. Continuous linear functionals and dual of locally convex spaces - Hahn- Banach separation theorems - Weak topology induced by a subset of the dual polar set – Bipolar theorem.
4. Weak* continuous linear functionals on dual normed linear spaces - Goldstein's theorem - Banach -Alaoglu's theorem - Characterization of reflexive spaces as spaces with weakly compact unit balls.
5. Linear operators – Examples - Integral operators - Inverse and adjoint operators - Adjoint operators in Hilbert spaces - Normal and unitary operators.

Reference books

1. Joseph Conway : A Course on Functional Analysis
2. H.H. Schefer: Topological Vector Spaces
3. M. Fabian, P. Habala, P. Hajek, V.M. Santalucia, J.Pelant and V. Zizle : Functional Analysis and Infinite Dimensional Geometry, CMS Books in Mathematics, Springer-Verlag, 2001.
4. B.V. Limaye : Functional Analysis
5. Thamban Nair : Functional Analysis

SOFT CORE : MATH-536 DIFFERENCE EQUATIONS

1. The Difference Calculus

Definition, Derivation of Difference equation, Existence and uniqueness theorem. Operators and E. Elementary difference operators. Factorial polynomials. Operators and the sum calculus. Examples.

2. First order difference equation

General Linear equation. Continued fraction. A general first-order equation – Expansion Techniques.

3. Linear Difference equations

Introduction, Linearly Dependent functions. Fundamental Theorem for homogeneous equations.

4. Inhomogeneous equations

In homogeneous equations. Second order equations. Sturm Liouville Difference equations.

5. Linear Difference equation with constant coefficients

Introduction, Homogeneous equation. Construction of a difference equation having specified solution. Relationship between Linear difference and differential equation.

Text book

Ronald E. Mickens : Difference equation - Theory and Application, Chapman & Hall, Second Edition, New York – London.

SOFT CORE : MATH-537 PARTIAL DIFFERENTIAL EQUATIONS

1. Partial differential equations of second order: Linear PDE with constant coefficients – Solutions – Classification of second order PDE – Solutions.
2. Laplace equation: families of equi-potential surfaces – Boundary value problems – Mean value and the maximum-minimum properties.
6. Separation of variables for Laplace equation
7. Wave equation- Elementary solutions – Riemann method for linear hyperbolic equation- Separation of variables.
8. Separation of variables for heat equation.

Text book

Treatment as in Ian N.Sneddon: Elements of Partial Differential Equations, McGraw-Hill International Edition, 1984.

- 1 – Chapter 3 (Sections 4, 5, 8, 9);
- 2 & 3 – Chapter 4 (Sections 2-5, 6, 8);
- 4 – Chapter 5 (Sections 2-4, 7);
- 5 – Chapter 6 (Sections 2-4, 6).

Reference book

Evans: Partial Differential Equations – Part I.

SOFT CORE : MATH-538
LIE GROUPS OF TRANSFORMATIONS AND DIFFERENTIAL EQUATIONS

1. Introduction–Lie groups of transformation – Infinitesimal transformations.
2. Extended group transformations and infinitesimal transformations (one independent – one dependent and two independent – two dependent).
3. Lie Algebras and Applications.
4. Invariance of first and second order differential equations.
5. Invariance of a partial differential equations of first and second order – elementary examples.

Text book

Treatment as in G. W. Blueman and S. Kumei: Symmetries and Differential Equations, Springer – Verlag, 1980.

- 1 – Chapter 2 (Sections 2.1 – 2.2);
- 2 – Chapter 2 (Sections 2.3.1 – 2.3.3) ;
- 3 – Chapter 2 (Sections 2.4.1 – 2.4.4);
- 4 – Chapter 3 (Sections 3.1.1 – 3.3.3);
- 5 – Chapter 4 (Sections 4.4.1 – 4.2.2).

SOFT CORE : MATH-539
NUMERICAL ANALYSIS FOR ORDINARY DIFFERENTIAL EQUATIONS

1. Euler's method - trapezoidal rule - theta method.
2. Adams - Bashforth method - Order and convergence - Backward Differentiation Formula.
3. Gaussian Quadrature - Explicit Runge - Kutta scheme - Implicit Runge Kutta scheme - Collocation.
4. Stiff equations - linear stability domain and A. Stability - A-stability of RK and multistep methods.
5. Error Control - Milne Device - Embedded Runge Kutta method.

Text book

Arieh Iserles, A First Course in the Numerical Analysis of Differential Equations, Cambridge.

SOFT CORE : MATH-540 ADVANCED FLUID MECHANICS

1. Characteristics - Wave equation - Examples - Riemann invariants - Hodograph transformation - Piston problem.
2. Shocks - Systems of conservation laws - Weak solution - Rankine - Hugoniot relations - Hugoniot relation - Prandtl's relation - Compressive shocks - Entropy condition.
3. Riemann problem - Centered waves - Solution of the Riemann problem - Courant – Fricdricts - Lewy condition.
4. Combustion waves - Single conservation law - Convex conservation laws - Oleinik's condition – Non convex systems of conservation laws - Solution.
5. Numerical methods - Finite Difference Methods- Forward Difference - Backward Difference - Central Difference - Consistency - Order - Stability - Lax's Theorem – Von Neumann Analysis - Godunov scheme - l_1 stability - l_2 stability - Lax – Fricdricks scheme - Lax Wendroff scheme - Crank - Nicolson scheme.

Reference books

1. Chorin and Marsden: A Mathematical Introduction to Fluid Mechanics, Texts in Applied Mathematics, Springer, Third Edition.
2. A Iserles : A First course in the Numerical Analysis of Differential Equations, Cambridge University Press.

SOFT CORE : MATH-541 INTEGRAL EQUATIONS

1. Introduction - classification of integral equation - examples
- IVP for ODE.
2. BVP for ODE - BVP for elliptic PDE - Abel's problem.
3. Second order ODE and integral equations -Differential equation theory - initial value problems - Boundary value problems - Singular Boundary value problems.
4. Integral equations of the second kind - Introduction - Degenerate kernels - a different approach.
5. Operators - Neumann series.

Text book

Porter and Stirling, Integral equations - Cambridge, pp 1-94.
A practical treatment from spectral theory to applications -
Cambridge: Cambridge University Press, 1996.

SOFT CORE : MATH-542 ADVANCED MATHEMATICAL ANALYSIS

1. Spaces of functions

Families of functions like periodic functions - Continuous functions, C^1 - functions, rapidly decreasing functions on \mathbb{R}_n which separate points, closed subsets - Partition of unity.

2. Topology on the spaces functions

Uniform convergence - Uniform convergence on compact on polynomials (with emphasis on power series), C^k -functions, C^1 -functions on \mathbb{R}_n – holomorphic functions on \mathbb{C} - Completeness of various spaces of functions under uniform metric, L_p -metric and under uniformly on compact topology.

3. Compact subsets

Arzela - Ascoli theorem - Normal families of holomorphic functions - Hilbert spaces of holomorphic functions - Reproducing kernels.

4. Fourier analysis

Convolutions - Fourier transform - Approximate identities in $L^1(\mathbb{R}_n)$ given by classical kernels like Fejer's kernel.

5. Density

Approximation through convolutions - Density theorems of Weierstrass and Stone, Korovkin – Density of C^1 -functions in L_p .

Reference books

1. R. Beals: Advanced mathematical analysis, Springer Verlag, New York, 1973.
2. J.B. Conway: Functions of one complex variable, Narosa Publishing House, 1980.
3. E.H. Lieb and M. Loss: Analysis, Narosa Book House, New Delhi, 1997.
4. W. Rudin: Real and complex analysis, 2nd ed., TMH Edition, 1962.
5. K. Yosida: Functional analysis, Springer - Verlag, New York, 1968.

SOFT CORE : MATH-543
REPRESENTATION THEORY OF COMPACT GROUPS

1. Locally compact groups - Examples of various matrix groups. - Existence of Haar measure (without proof) - Computation of Haar measure on \mathbb{R} , \mathbb{T} , $SU(2)$, $SO(3)$ and some simple matrix groups. Convolution - The Banach algebra $L^1(G)$.
2. General properties of representations of a locally compact group - Complete reducibility - Basic operations on representations – Irreducible representations.
3. Representations of finite groups - Decomposition of regular representations – Orthogonal relations - Irreducible representations of the symmetry group. Representations of compact groups - Matrix coefficients - Schur's orthogonality relations - Finite dimensionality of irreducible representations of compact groups.
4. Arzela - Ascoli Theorem - Compact operators - Various forms of Peter- Weyl theorem.
5. Character of a representation. Schur's orthogonality relations among characters - Weyl's character formula - Computing all the irreducible representations of $SU(2)$, $SO(3)$.

Reference books

1. T. Brocker and T. Dieck: Representations of compact lie groups, Springer Verlag, 1985.
2. J. L. Clerc, Les repr´esentations des groupes compacts, Analyse harmonique (J.L.Clerc et al. ed.), C.I.M.P.A, 1982.
3. M. Sugiura, Unitary representations and harmonic analysis: An introduction, John Wiley, 1975.
4. B. Vinberg, Linear representations of groups, A series of advanced textbooks in Mathematics, Birkhauser Verlag, Berlin - Boston, 1989.

SOFT CORE : MATH-544 ELEMENTS OF HARMONIC ANALYSIS

1. Basic properties of topological groups, subgroups, quotient groups - Examples of various matrix groups.
2. Connected groups - Existence of Haar measure (without proof) - Computation of Haar measure on \mathbb{R} , \mathbb{T} , \mathbb{Z} and some simple matrix groups - Convolution, the Banach algebra $L^1(G)$ and convolution with special emphasis on $L^1(\mathbb{R})$, $L^1(\mathbb{T})$ and $L^1(\mathbb{Z})$.
3. Fourier transform and its properties - Approximate identities in $L^1(G)$.
4. The Dual group of a locally compact abelian group - Computation of dual groups for \mathbb{R} , \mathbb{T} , \mathbb{Z} .
5. Classical kernels on \mathbb{R} - The Fourier inversion Theorem - Plancherel theorem on \mathbb{R} - Plancherel measure on \mathbb{R} , \mathbb{T} , \mathbb{Z} - Discussion on Plancherel theorem on a general locally compact abelian group.

Reference books

1. G. Folland: A course in abstract harmonic analysis, CRC Press, 1994.
2. H. Helson: Harmonic analysis, Trim Series, Hindustan Book Agency, 2nd Edition, 1995.
3. Y. Katznelson: Introduction to harmonic analysis, J. Wiley and Sons, 1968.
4. L.H. Loomis: An introduction to abstract harmonic analysis, van Nostrand, New York, 1953.
5. E. Hewitt & K.A. Ross: Abstract harmonic analysis, Vol. I, Springer – Verlag, 1963.
6. W. Rudin: Real and complex analysis, Tata Mc Graw Hill, 2nd Edition, 1962.

SOFT CORE : MATH-545 LINEAR LIE GROUPS

1. Basic properties of topological groups, subgroups, quotient groups and connected groups.
2. Linear Lie groups like $GL(n, \mathbb{R})$, $GL(n, \mathbb{C})$, Orthogonal groups, Unitary groups, Motion groups, Heisenberg groups and various properties of them.
3. Computation of Haar measure for the above groups - The exponential map and the Lie algebras of the above groups.
4. Representations of a locally compact group - Adjoint representation - Irreducible representations of $SU(2)$ and $SO(3)$.
5. Induced representation - Irreducible representations of Motion group $M(2)$ and Heisenberg groups.

Reference books

1. J. L. Clerc: Les représentations des groupes compacts, Analyse harmonique (J.L.Clerc et al., ed.), C.I.M.P.A., 1982.
2. G. Folland: A course in abstract harmonic analysis, CRC Press, 1994.
3. S. Kumaresan: A course in differential geometry and lie groups, Trim 22, Hindustan Book Agency, 2002.
4. M. Sugiura: Unitary representations and harmonic analysis: An introduction, John - Wiley, 1975.

SOFT CORE : MATH-546 GRAPH THEORY

1. Definition of a Graph-Varieties of Graphs-Walks and Connectedness-Degree of vertex-External Graphs-Intersection of Graphs-Operation on Graphs - Cutpoints, Bridges and blocks-Block Graphs and cutpoint Graphs.
2. Characterization of trees-Centres and Centroids-Block-Cut point trees - Connectivity and line Connectivity.
3. Eulerian Graphs - Hamiltonian Graphs.
4. Plane and Planar Graphs-outerplanar graphs-Kuratowski's theorem - other characterization of planar graphs- Thickness and crossing number.
5. The chromatic number-The Five colour theorem-The Four colour conjecture.

Text book

Treatment as in F. Harray: Graph Theory.

SOFT CORE : MATH-548 ADVANCED TOPICS IN DISCRETE MATHEMATICS

1. Applications of Boolean algebra to Switching Theory using AND, OR and NOT gates - The Karnaugh Method.
2. Definition of (undirected) graphs - Paths - Circuits and Cycles - Subgraphs and induced subgraphs - Degree of a vertex -Connectivity - Complete graphs bipartite graphs - Matrix representations of graphs - Weighted graphs - Dijkstra's Algorithm.
3. Trees and their properties- Spanning trees - Minimal spanning trees - Kruskal's Algorithm – Euler graphs and paths - Euler's theorem on the existence of Euler paths and circuits.
4. Fundamental cycles - Cutsets - Fundamental cutsets - Plane and planar graphs - Dual graphs - Euler's formula for connected plane graphs - Kuratowski's theorem (statement only) and its applications.
5. Directed graphs - Indegree and outdegree of a vertex - Strong connectivity - Matrix representation of directed graphs - Warshall's Algorithm - Directed trees. Search trees - Tree traversals- Notions of syntax analysis- Polish notations - Conversion of infix expressions to Polish notations - The reverse Polish Notation.

Reference books

1. J.P. Tremblay and R. Manohar, Discrete Mathematical Structures with Applications to Computer Science, McGraw Hill Book Company, 1997.
2. J.L. Gersting, Mathematical Structures for Computer Science (3rd edn), Computer Science Press, New York.
3. Seymour Lipschutz, Finite Mathematics, McGraw - Hill Book Company, New York, International Edition, 1983.
4. S. Wiitala, Discrete Mathematics - A Unified Approach, McGraw Hill Book Company.
5. C.L. Liu, Elements of Discrete Mathematics, McGraw-Hill Book Company.
6. N. Deo, Graph Theory with Applications to Engineering and Computer Sciences, Prentice Hall of India.

SOFT CORE : MATH-549 LABORATORY PRACTICAL IN MATHEMATICS

Objective of the Course:

To introduce to the students certain important software packages in Mathematics and train them for practical applications so as to augment the skills of the students.

Course contents: Any five units from the following topics:

Kash
Latex
Maple
Mathematica
Mathcad
Mathlab
Octave
Pari
Reduce
Scilab

Methodology of teaching:

Lectures
Computer Practicals
Assignments

Methodology of evaluation: Practicals

Reference books

Adre Heck : Introduction to Maple, Springer, 2003.

Batut C, Bernardi D, Cohen H and Olivier M: User's Guide to PARI – GP, University of Bordeaux I, France, 1995.

Hearn: REDUCE, Springer.

Wolfram S: Mathematica.

SOFT CORE: MATH 550 TOPICS IN TOPOLOGY AND ANALYSIS

1. Connected topological spaces

Connected topological spaces- Path connectedness- -Components-Example of a connected but not path connected space- Locally connected spaces- Continuous images of connected, path connected and locally connected spaces

2. Locally compact spaces and Completely regular spaces

Locally compact spaces-One point compactification -Completely regular spaces- Imbedding theorem for completely regular spaces-The Uryshon's metrization theorem

3. Stone- Cech compactification

Stone- Cech compactification- equicontinuity- Arzela- Ascoli theorem- Stone-Weierstrass theorem

4. Interpolation

Lagrange interpolation-Bernstein polynomials- Monotone operator theorem- Weierstrass theorem

5. Completeness

Completeness of $L_p[a; b]$ -Product measures- Fubini's theorem

Reference books

James R Munkres: Topology

Walter Rudin: Real analysis

H.L.Royden: Real Analysis

E.W.Cheney: An Introduction to Approximation Theory, Chelsea Publication

SOFT CORE: MATH 551 FUNCTIONAL ANALYSIS - II

1. Nets and sequences – Characterization of topological properties in terms of nets – Inadequacy of sequences – Subnets – Unconditionally summable series – Examples – Topological vector spaces – Balanced and absorbing sets – Bounded sets – Locally convex spaces.
2. Weak topology generated by a subset of the dual of a normed linear space and continuous linear functionals - Weak and weak* topologies - Weak and weak* convergent sequences – Comparison of weak, weak* and norm topologies on finite and infinite dimensional normed linear spaces – Minkowski functional – Sublinear functional and their continuity – Semi norms - Characterization of normable locally convex space.
3. Duality formula – Duals of subspaces and quotient spaces, of normed linear spaces – Hahn-Banach separation theorems in a normed linear space.
4. Weak* continuous linear functionals – Goldstein's theorem – Banach-Alaoglu's theorem - Characterization of reflexive spaces as spaces with weakly compact unit balls – Closed subspaces of reflexive spaces are reflexive.
5. The spaces l_p and $L_p[a, b]$ are reflexive for $1 < p < \infty$ - Completeness of $L_p[a, b]$ for $p \geq 1$ – Dual of $L_p[a, b]$ for $1 \leq p < \infty$.

Reference books

1. J. Conway: A Course on Functional Analysis
2. M. Fabian, P. Habala, P. Hajek, V.M. Santalucia, J. Pelant and V. Zizler: Functional Analysis and Infinite Dimensional Geometry, CMS Books in Mathematics, Springer-Verlag Seminar Volume, 2001.

SOFT CORE: MATH 552 OPERATOR THEORY

1. Banach algebras – Involutive Banach algebras – Various examples including Group algebras – Spectrum – Spectral mapping theorem – Spectral radius formula.
2. Maximal ideal space for commutative Banach algebras – Gelfand - Naimark theory for commutative Banach algebras – C*- algebras, Examples- Commutative C*- algebras.
3. Representations of C*-algebras – Von Neumann's density theorem – Double commutant theorem - GNS constructions.
4. Functional calculus – The spectral theorem for normal operators – Spectral theorem for unitary operators – Polar decomposition.
5. Compact operators – Examples and properties – Spectral theorem for compact operators – Hilbert – Schmidt operators.

Reference books

Sunder, V.S: Functional Analysis – Spectral Theory, Trim Series, Hindustan Book Agency, 1997.

Takesaki, M: Theory of Operator Algebras I, Springer Verlag, 1979.

Yosida, K: Functional Analysis, Springer Verlag, 1968.

SOFT CORE: MATH - 554
NON-COMMUTATIVE RINGS AND REPRESENTATIONS

1. Modules

Modules - Artinian and Noetherian modules - Tensor products - Restricted and induced modules - Indecomposable modules – Completely reducible module – Schur Lemma.

2. Radical

Semi simple rings - The radical of a rings - The Jacobson radical – Group algebras - Maschke's Theorem.

3. Structure theory

Structure theory of ring - Density Theorem - Wedderburn-Artin theorem for semi simple rings.

4. Representations – I

Representations - linear representation - Matrix representation - Equivalent representation - Invariant subspaces - Irreducible representations

5. Representations – II

Direct sum of representations - Induced representation – restricted representation - tensor product of representations - Inner products of representation.

Text book

Charles W. Curtis and Irving Reiner: Representation Theory of Finite Groups and Associative Algebras, Inter Science Publishers, 1962. (Chapters 2 and 4).

Reference books

1. William Fulton and Joe Harris: Representation Theory - A First Course, Springer International Edition, Springer-Verlag, New York, 2004.
2. Jacobson: Basic Algebra II, Hindustan Publishing Corporation (India), 1983.
3. I.N Herstein: Non-Commutative Rings, Mathematical Association of America.

SOFT CORE: MATH 555 COMPLEX ANALYSIS

1. The space of continuous functions - Spaces of analytic functions - Spaces of meromorphic functions - The Riemann Mapping Theorem.
2. Weierstrass factorization theorem - Factorization of sine function - The gamma function - The Riemann zeta function.
3. Schwarz Reflection Principle - Analytic continuation along a path - Monodromy theorem.
4. Subharmonic and superharmonic functions - The Dirichlet problem - Green's function.
5. Jensen's formula - The genus and order of an entire function.

Text book

John B. Conway: Functions of One Complex Variable, Second edition, 1980.

SOFT CORE: MATH-556 ALGORITHMS USING C++

Objectives of the Course:

To make the students familiar with certain mathematical algorithms and their implementation through C++ language and equip them well with hands-on experience to acquire skills in solving mathematical problems using computers.

Fundamentals of algorithms

1. Introduction to algorithms – Steps in the development of algorithms – Examples of algorithms which are significant from computational point of view.

Fundamentals of C++ language

2. Constants- Variables - Declaration of variables - Type conversions - Relational operators - Decision making, branching and looping. Functions - Simple functions - Passing arguments to functions - Returning values from functions - Reference arguments - Overloaded functions - Inline functions.

3. Defining classes - Creating objects - Constructors - Accessing class members - Member functions - Overloaded constructors - Static class data - Arrays and string functions..

4. Operator overloading - Overloading unary and binary operators- Data conversion - Derived class- Class hierarchies – Inheritance - Public and private inheritance – Types of inheritance. Pointers - Pointer to objects - Memory management - New and delete functions.

5. Applications of C++ in algorithms

Development of algorithms using C++ - Applications to solve mathematical problems. Sorting algorithms - Bubble sort – Selection sort – Insertion sort. Searching algorithms - Binary search – Linear search.

Reference books

A.V.Aho, J.E.Hopcroft and J.D.Ullman, The Design and Analysis of Computer Algorithms, Addison-Wesley, Reading, Mass., 1974.

R.G.Dromey: How to solve it by computer, Prentice-Hall of India Private Limited, New Delhi, 1999.

Robert Lafore: Object Oriented Programming in Turbo C++, Galgotia Publishers Private Ltd, 1997.

SOFT CORE: MATH-557 ALGORITHMS USING JAVA

1. ALGORITHMS:

Introduction : Data structure concepts-List, Stacks, Queues, Trees, Heaps, Sets, Graphs, Design of Efficient Algorithms and their Computational Complexities . Searching and sorting techniques.

2. JAVA:

Data types – Constants, variables – Declaration of variables – Scope of variables- Types casting –operator expressions- Decision making , branching and looping , creating arrays, variable size arrays, Strings.

3. Defining classes-Creating objects-Accessing class members – Methods overloading, Creating inheritance – Abstract methods and classes.

4. Multi threading programming – creating threads, Extending the thread class – Starting , Stopping and blocking threads-Managing errors and exceptions-Types of errors exceptions-Catch statements-Throwing customized Exceptions.

5. Applet programming-Building Applet codes-Adding Applet to HTML file – passing parameters to Applet.

Text books

1. Alhred V. Aho, Jellrey D. Ullman, John E.Hop Croft, “Data Structures and Algorithms “, Addison Wesley Series.
2. E.Balagurusamy : “Programming with Java: A Primer”, Tata McGraw Hill. 1998.

Reference books

1. Ellis Horowitz and Sartaj Sahani, “ Fundamentals of computer Algorithms”.
2. Gilles Brassard and Paul Bratley, “Fundamentals of Algorithms”, Prentice Hall of India Pvt. Ltd., 1997.

SOFT CORE: MATH-558 FUNCTIONAL ANALYSIS - III

1. Nets and sequences – Convergence of nets – Unconditional summability of series in Banach spaces – Review of Hilbert spaces – Sesquilinear forms – Adjoint and self-adjoint operators in Hilbert spaces.
2. Normal and unitary operators - Weak topology – Strong and weak convergence of operators – Orthogonal direct sums.
3. Compact operators of Banach spaces – Integral operators - Adjoint operators – Fredholm alternative.
4. Invertible operators – Eigenvalues and spectrum related results – Spectrum of a compact operator - Self-adjoint operators.
5. Compact operators of Hilbert spaces – Numerical range – Spectral theorem for compact, self-adjoint operators.

Text books

1. V.S.Sundar: Functional Analysis, TRIM Series 13, Hindustan Book Agency (for units 1 and 2).
2. M. Fabian, P.Habala, P. Hajek, V.M. Santalucia, J.Pelant and V. Zizler: Functional Analysis and Infinite Dimensional Geometry, CMS Books in Mathematics, Springer-Verlag Seminar Volume, 2001 (for units 3-5).

